PHY209 Electromagnetism
Assignment 4

Handed out: September 5, 2019

Problem 1

Express the cylindrical unit vectors $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\phi}}$, $\hat{\boldsymbol{z}}$ in terms of $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$. Invert your formulas to get $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ in terms of $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\phi}}$, $\hat{\boldsymbol{z}}$ (and ϕ).

Problem 2

Find the potential on the axis of a uniformly charged solid cylinder, a distance z from the center. The length of the cylinder is L, its radius is R, and the charge density is $\rho^{'}$. (Assume that $z > \frac{L}{2}$.) Use your result to calculate the electric field at this point.

Problem 3

(a) Consider a half-infinite hollow cylindrical shell (that is, one that extends to infinity in one direction) with radius R and uniform surface charge density σ. What is the electric field at the midpoint of the end face?
(b) Use your result to determine the field at the midpoint of a half-infinite solid cylinder with radius R and uniform volume charge density ρ, which can be considered to be built up from many cylindrical shells.

Problem 4

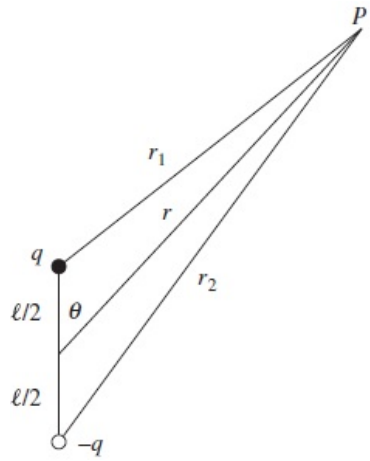
Use the result in Problem 3(a) to find the field along the axis of a long hollow cylindrical tube (that extends to infinity in both directions), which carries a uniform surface charge σ.

Problem 5

(a) Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$ (the magnitude of the position vector).
(b) Find the gradient of the function $f(x, y, z) = x^2 + y^3 + z^4$.
(c) Let $\boldsymbol{r}$ be the separation vector from a fixed point $(x^{'}, y^{'}, z^{'})$ to the point (x, y, z), and let r be its length. Show that $\nabla(r^2) = 2\boldsymbol{r}$.

Problem 6

An electric dipole consists of two equal and opposite charges ($\pm q$) separated by a distance l. In the limit $r \gg l$, find the approximate potential at point P in terms of r, l and θ only. Hint: use the law-of-cosines expressions for r_1 and r_2.



Problem 7

An electric dipole of dipole moment $p = qd$, is situated at the origin, pointing in the z direction. Assume $a \gg d$.
(a) What is the force on a point charge q at $(a, 0, 0)$ (Cartesian coordinates)?
(b) What is the force on q at $(0, 0, a)$?
(c) How much work does it take to move q from $(a, 0, 0)$ to $(0, 0, a)$?

Problem 8

Consider a dipole $\boldsymbol{p}$ at the origin, pointing in the z direction. For any far away point P in the xz plane, find the components of the electric field in the direction of $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{z}}$. Also, find the components in the direction of $\hat{\boldsymbol{r}}$ and the direction perpendicular to $\hat{\boldsymbol{r}}$. Hint: project the vector (E_x, E_z) onto the radial and tangential directions.